Efficient Testing Combining Design of Experiment and Learn-to-Fly Strategies

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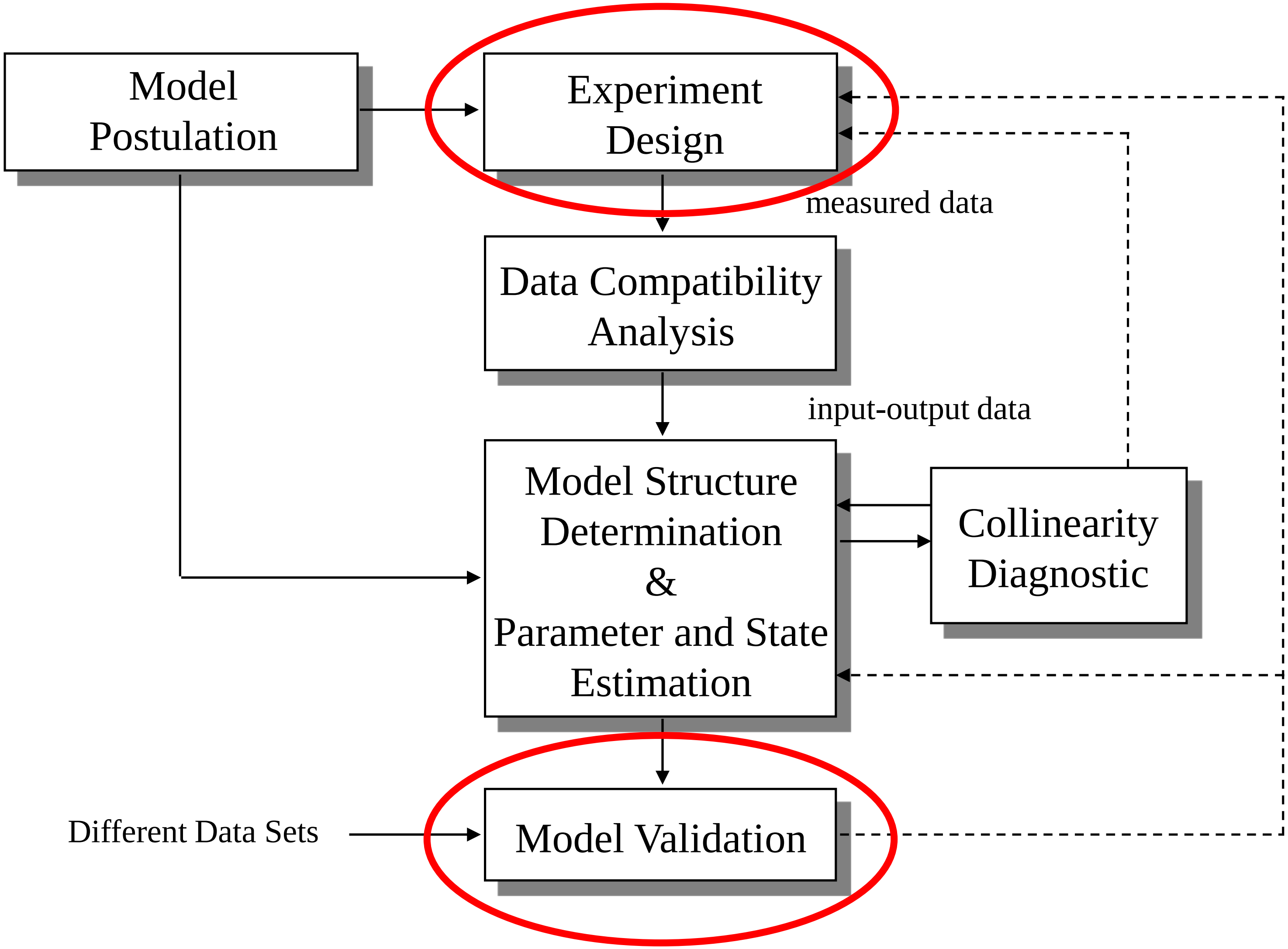
Outline

- Introduction
  - Seeking greater efficiency & performance through experiment design
    - Efficiency gained by collecting the “right amount” of data
    - Performance gained by adding statistical rigor
- System Identification Process in Wind Tunnel
  - Design of Experiment (DOE)
  - Learn-to-Fly (L2F)
  - Blended DOE-L2F
    - First time testing blended concept – strawman approach
    - Work in progress
- Analysis, Results, and Validation Tests
  - DOE Tests
  - L2F Tests
  - Blended DOE-L2F Tests
- Concluding Remarks

Motivation: Seek Efficiency Using Experiment Design

- Wide spectrum of modeling demands
  - Fidelity requirements
  - Aircraft complexity
- Aircraft complexity drive up costs
  - Conventional practice in LaRC 12-foot Wind Tunnel (static test)
    - 100 Hz sample rate, dwell for 10 seconds, average data
    - ~ 2 data pts/min
  - Simple factorial test for L-59
    - 9-Factors: α, β, and 7 control surfaces
    - 2^9 = 512 test points => 4.26 hours
  - Reasonable data density often requires 5^9 => 16,276 hours (~8years)!
- Investigators must tradeoff of cost vs fidelity/complexity
  - Define purpose of model and required fidelity. What is allowable error?
  - Asking for “best possible answer” is not adequate
  - Speeding up the modeling process helps anywhere on spectrum

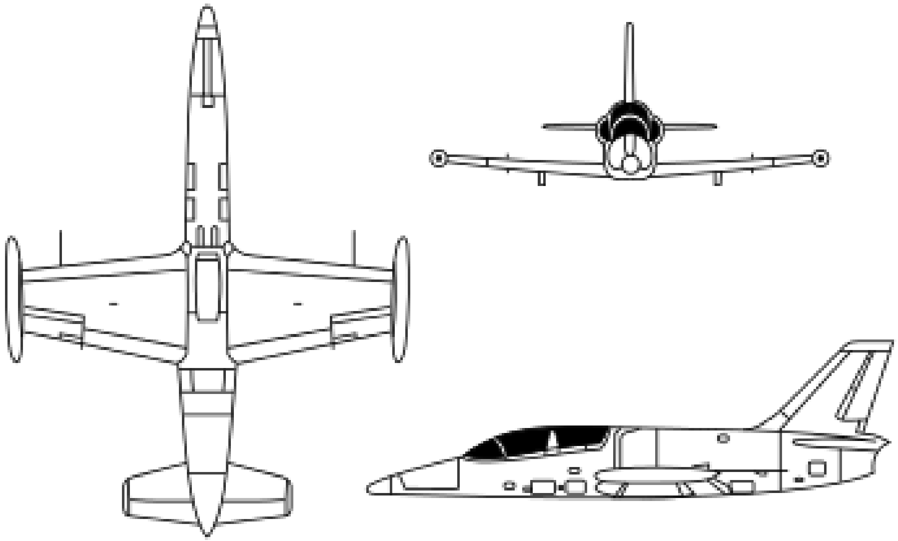
Aircraft System Identification Process



Test vehicle for Wind Tunnel Static Test

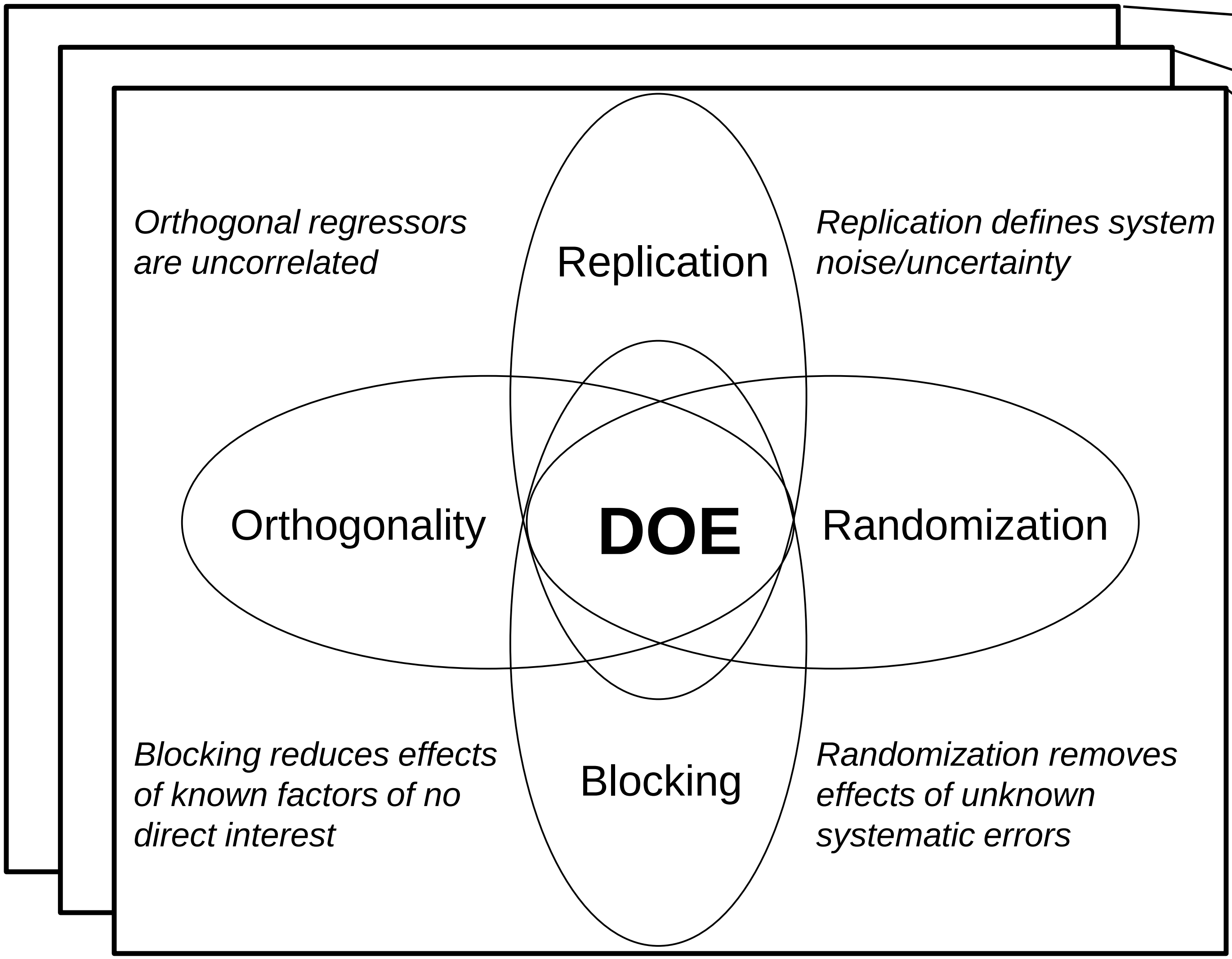
No.	Label	Description	Low Value	High Value	Units
1	aoa	Aircraft alpha	-2	20	deg
2	beta	Aircraft beta	-5	5	deg
3	dela_L	Aileron left wing	-25	25	deg
4	dela_R	Aileron right wing	-25	25	deg
5	delf_L	Flap left wing	0	40	deg
6	delf_R	Flap right wing	0	40	deg
7	delr	Rudder	-30	30	deg
8	dele_L	Elevator left wing	-30	30	deg
9	dele_R	Elevator right wing	-30	30	deg

- L-59 Albatros
- Czech military trainer
- Low-cost off-the-shelf kit
- 12.5% scale model
- Sport application, RC actuators



Tenets of Design of Experiment (DOE)

Sequential testing proceeds only as model complexity requires



Block 1 validation

Block 2 validation

Block 3 validation

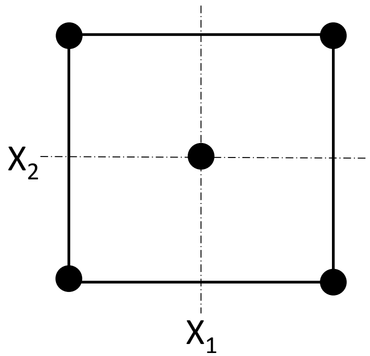
…

Blocks of data, collected as needed, each followed by model validation

Block Designs & Supported Models

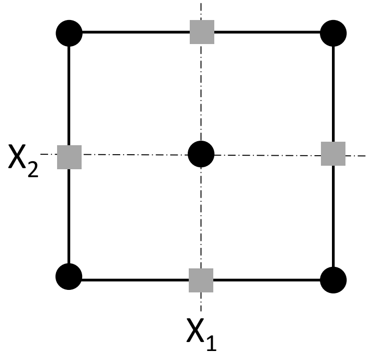
- Full factorial design

$$y = B_0 + \sum_i B_i x_i + \sum_{i \neq j} \sum B_{ij} x_i x_j + \varepsilon \qquad i = 1, 2, \ldots, k$$



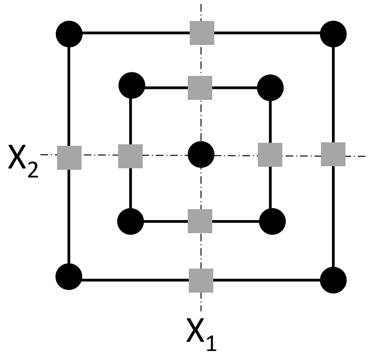
- Face-centered design (FCD)

$$y = B_0 + \sum_i B_i x_i + \sum_i B_{ii} x_i^2 + \sum_{i \neq j} \sum B_{ij} x_i x_j + \varepsilon \qquad i = 1, 2, \ldots, k$$



- Nested face-centered design

$$y = B_0 + \sum_i B_i x_i + \sum_i B_{ii} x_i^2 + \sum_{i \neq j} \sum B_{ij} x_i x_j + \sum_i B_{iii} x_i^3 + \varepsilon \qquad i = 1, 2, \ldots, k$$



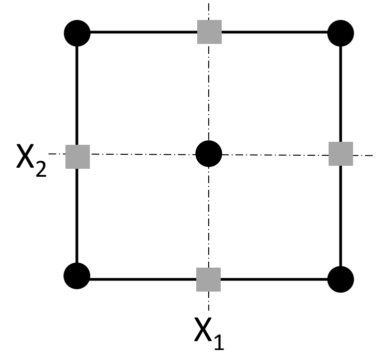
Block 1, DOE Design Metrics (9-factors)

Block Type	Blocks	Runs	Design Terms	VIF	% Power
	(inclusive)				2σ, s/n=2
¼ Fraction FCD	1	156	Quadratic	9.64	84.4

Maximum Variance Inflation Factor (VIF),
reflects lack of orthogonality in design; desire ≤ 10

% Power reflects statistical power of design,
manages type-2 error; desire ≥ 80

$$y = B_0 + \sum_i B_i x_i + \sum_i B_{ii} x_i^2 + \sum \sum_{i \neq j} B_{ij} x_i x_j + \varepsilon \qquad i = 1, 2, \ldots, k$$

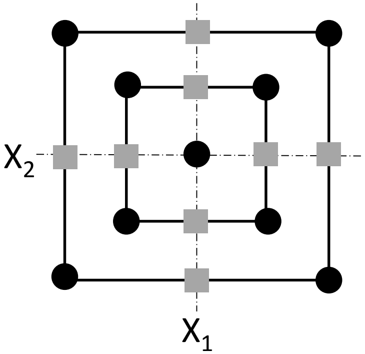


Validation Test Performed after each block of data

Block 2 added to create Nested FCD

Block Type	Blocks	Runs	Design Terms	VIF	% Power
	(inclusive)				2σ, s/n=2
¼ Fraction FCD	1	156	Quadratic	9.64	84.4
Nested FCD	1, 2	312	Quadratic	22.41	86.8

$$y = B_0 + \sum_i B_i x_i + \sum_i B_{ii} x_i^2 + \sum \sum_{i \neq j} B_{ij} x_i x_j + \sum_i B_{iii} x_i^3 + \varepsilon \qquad i = 1, 2, \ldots, k$$



Require optimized design points to reduce VIF

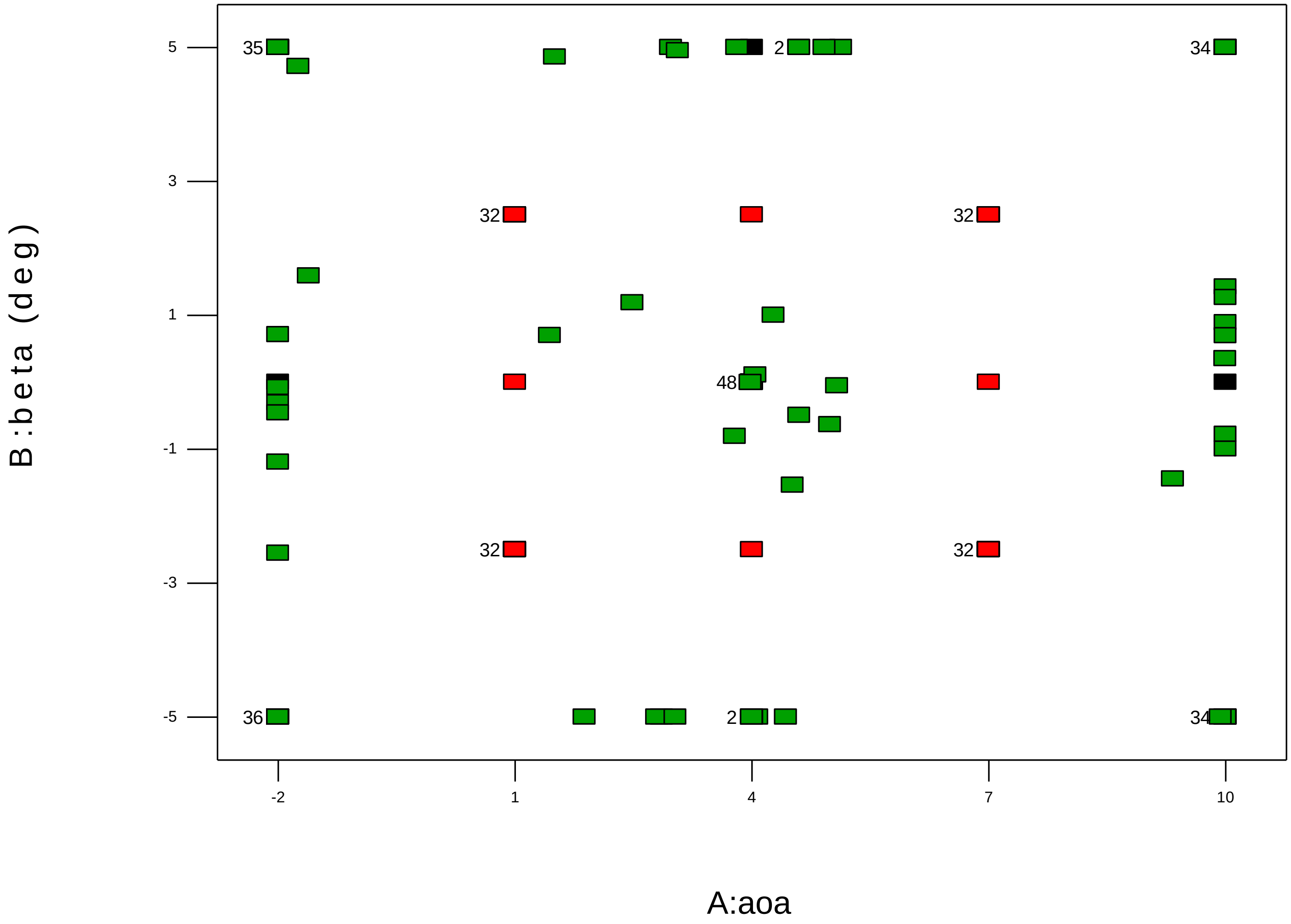
Final DOE Design, 3-blocks

Block Type	Blocks (inclusive)	Runs	Design Terms	VIF	% Power 2σ, s/n=2
¼ Fraction FCD	1	156	Quadratic	9.64	84.4
Nested FCD	1, 2	312	Quadratic	22.41	86.8
I-optimal	1, 2, 3	366	Quadratic	4.0	99.9

I-optimal block provides test points that minimize prediction error

Validation Test Performed after each block of data

DOE Design for 3 blocks



- FCD (black)
- Nested FCD (red)
- I-optimal (green)

Stepwise Regression Modeling

- Stepwise regression used to select model parameters

$$y = \beta_0 + \sum_i \beta_i x_i + \sum_i \beta_{ii} x_i^2 + \sum_{i \neq j}\sum \beta_{ij} x_i x_j + \sum_i \beta_{iii} x_i^3 + \ldots + \varepsilon \quad i = 1, 2, \ldots, 23$$

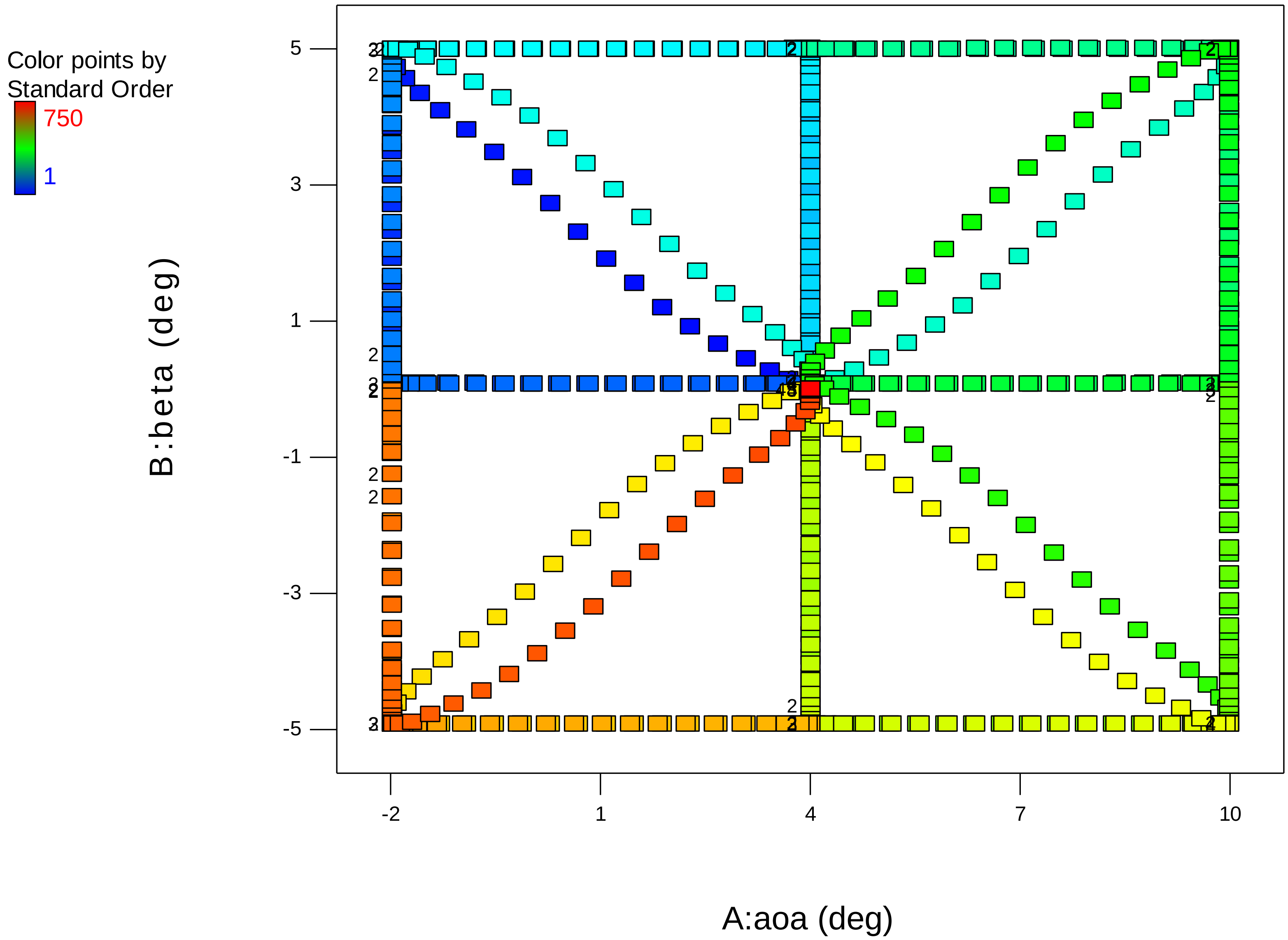
- Primary metrics utilized for model selection:
  - Stepwise Regression significance level: 95% – 99%
  - Standard ANOVA table analysis
  - Lack of Fit (LOF) measure of model error relative to pure error
  - Standard deviation (fit error)
  - PRESS (prediction error sum of squares)
  - Coefficient of Variation (C.V.% = std. dev. / mean)
  - e_i / $C_N$max %; (e_i = C_N_measured – C_N_predicted) … desire ≤ 3%
  - R^2 , adjusted R^2 , predicted R^2 , (family of metrics)

$$R^2 = \frac{\text{variation explained}}{\text{total variation}}; \quad 0 < R^2 < 1$$

Learn-to-Fly (L2F) Testing

- L2F approach adapted to wind tunnel
  - General L2F approach is real-time global modeling of aerodynamics
  - Applicable to wind tunnel or flight testing
  - Continuous sampling during dynamic test
- This study is a "quasi-static" test
  - Continuous sampling while sweeping target points slowly
  - Batch processing, stepwise regression
- Key to efficiency: Wide-band orthogonal inputs
  - Higher bandwidth (HBW) inputs applied to control surfaces
  - Lower bandwidth (LBW) inputs apply to other factors
- L2F experiment design
  - Test grid is setup for LBW factors
  - LBW trajectories form a nested "FCD-like" design

Learn-to-Fly (L2F) Trajectories



Blended DOE-L2F Testing ("quasi-static" test)

- Use key "efficiency features" of both approaches
  - DOE: 4 tenets, sequential testing blocks of data, with validation tests
  - L2F: HBW design for factors that accept wide-band inputs
- Blended design both simplifies and complicates final design
  - Simplifies 9-LBW experiment to a 2-LBW + 7-HBW experiment
  - Complicates evaluation of design metrics
- Strawman blended design
  - Design for 9-LBW experiment ensure all factors are included in design
  - Keep statistical advantages and design metrics of DOE
  - Assume "extra" data between target points enhances modeling
  - Assume blended design is obtained by removing redundant $\alpha-\beta$ targets
  - Blended designs require rig move slow enough to allow full sweep of controls at each $\alpha-\beta$ target point

Blended DOE-L2F Design Metrics (9-factors)

Block Type	Blocks included	Runs target points	Terms	VIF	% Power 2σ, s/n=2
Factorial	1	134	Linear + 2FI	1*	99.7
FCD	1, 2	156	Quadratic	9.68	84.2
Nested FCD	1, 2, 3	312	Quadratic	22.47	86.7
I-optimal	1, 2, 3, 4	384	Quadratic	4.85	99.9

*Squared factors are aliased

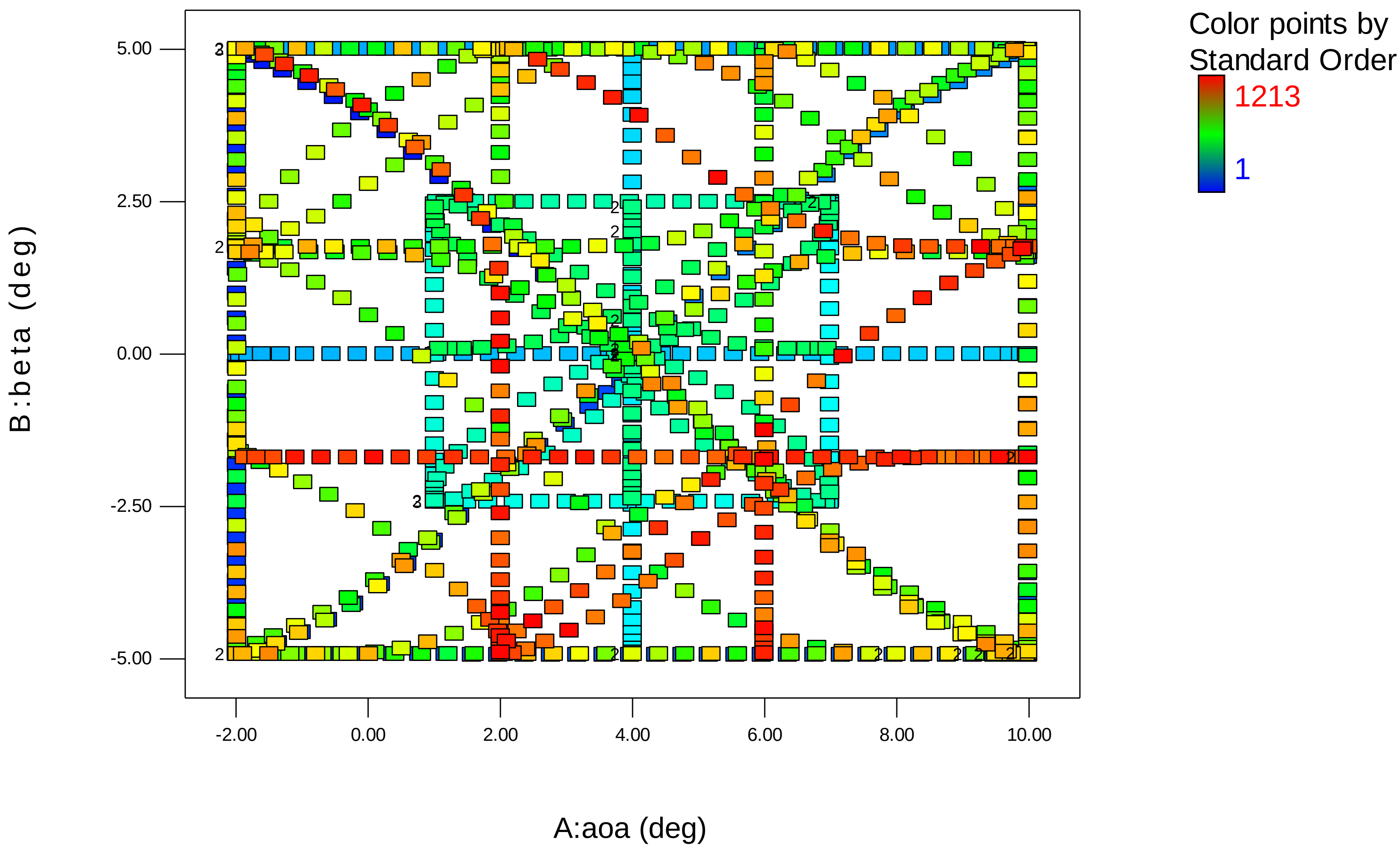
Some Lessons Learned:

Fewer blocks required with continuous sampling

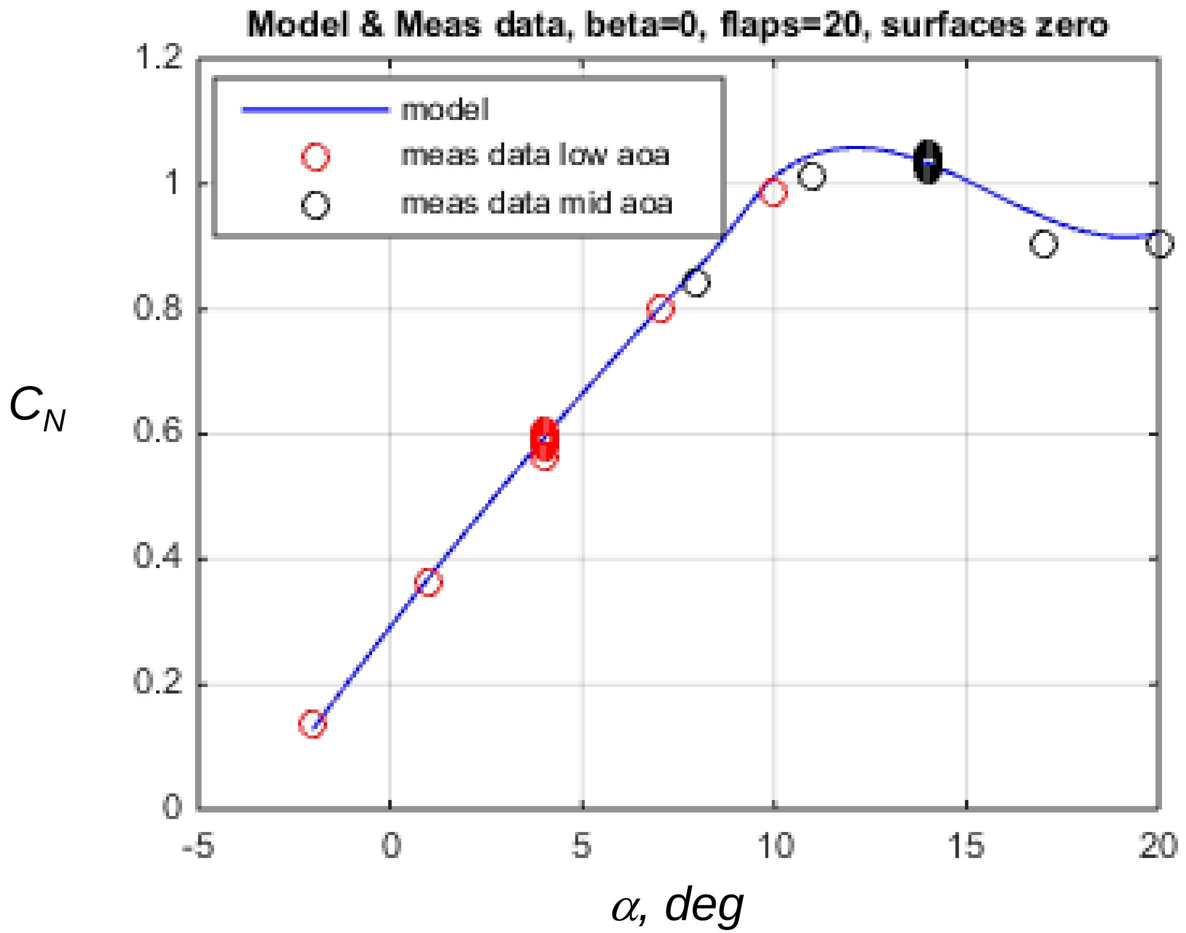
Divide optimal blocks!

4th block provided too much data for the blended design.

Blended DOE-L2F Trajectories

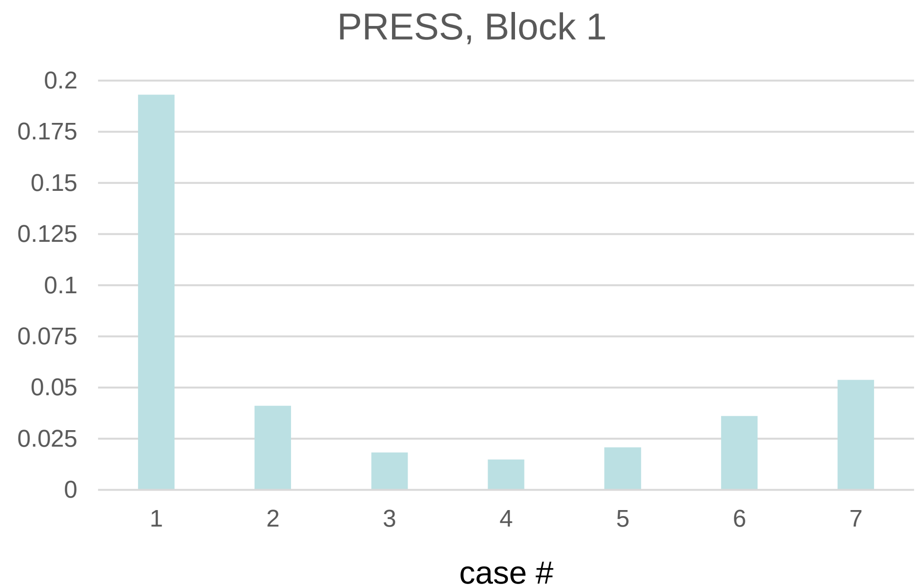


DOE Model (3 blocks)



DOE Modeling Progression

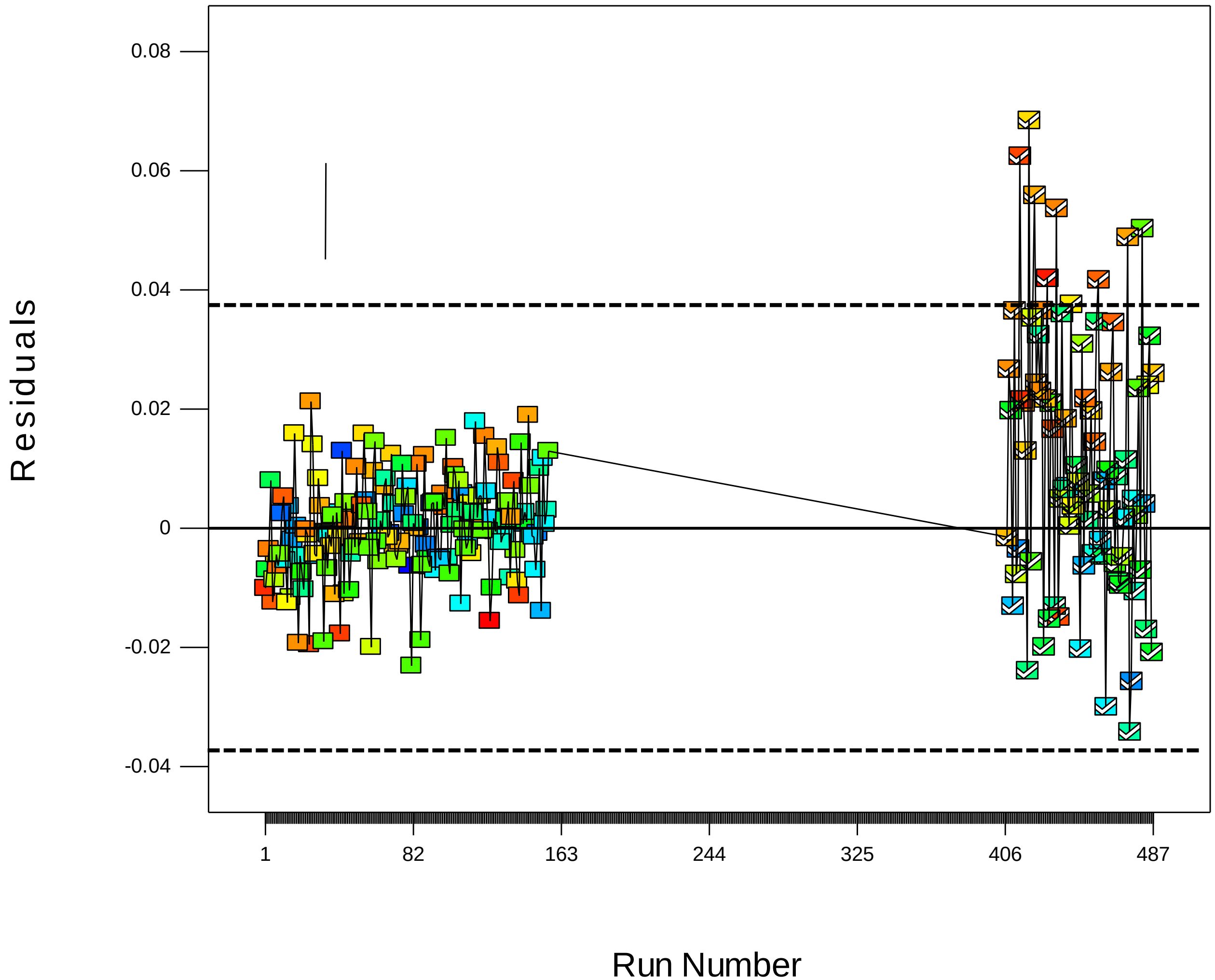
- Block 1 (FCD)
- 1st in series of sequential tests
- Case #2 – error budget satisfied
- Case #3 – best model is cubic
- Case #4 – minimum PRESS
- Case #6 – minimum Std. Dev



case #	1	2	3	4	5	6	7
block #	1	1	1	1	1	1	1
Design model	FCD	FCD	FCD	Nested FCD	I-Optimal	I-Optimal	I-Optimal
Model terms (No.)	Linear + 2FI (12)	quadratic + 2FI (20)	cubic + 3FI (32)	cubic + 3FI (38)	cubic + 3FI (68)	cubic + 3FI (81)	cubic + 3FI (128)
R^2	0.9931	0.9988	0.9996	0.9997	0.9995	0.9996	0.9999
Std. Dev.	0.0351	0.0149	0.0095	0.0084	0.0064	0.0060	0.0060
PRESS	0.1931	0.0411	0.0183	0.0149	0.0208	0.0362	0.0538
**e_i/C_Nmax %	5.68%	0.22%	0.15%	0.11%	0.12%	0.10%	0.10%

*residual $e_i = C_N$_measured – C_N_predicted, **$C_N$max = 1.22

Validation Test, DOE Block 1



- Residuals vs Run
- Block 1, ¼ FCD
- C_N low α range
- Case #3 model
- 8 fail 3% error

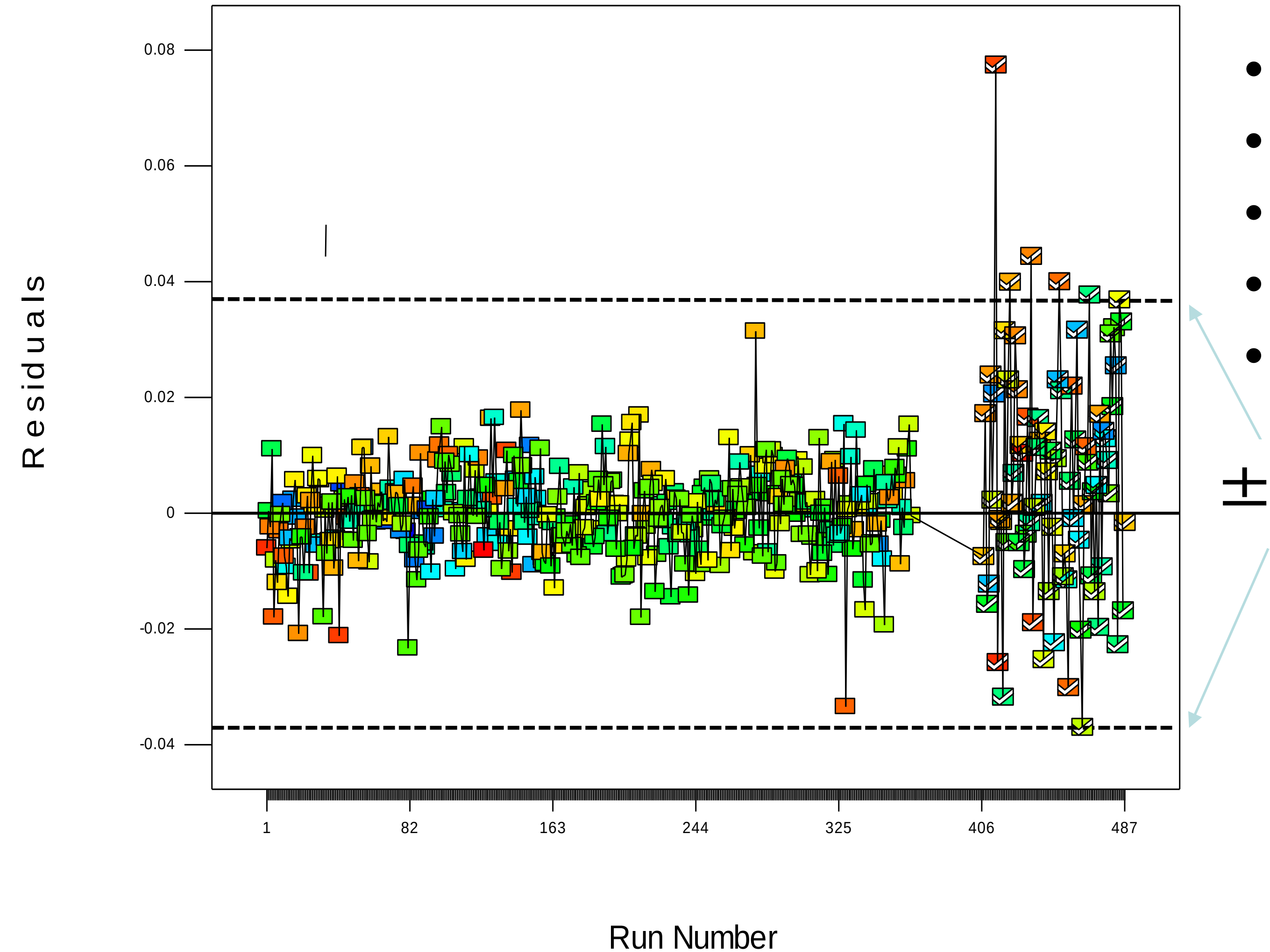
$\pm$3 % error

<u>$\pm$3% error test</u>

- 8 points failed
- 9 of 81 allowed

Validation tests reveal true prediction & bias errors

Validation Test, DOE Blocks 1-3

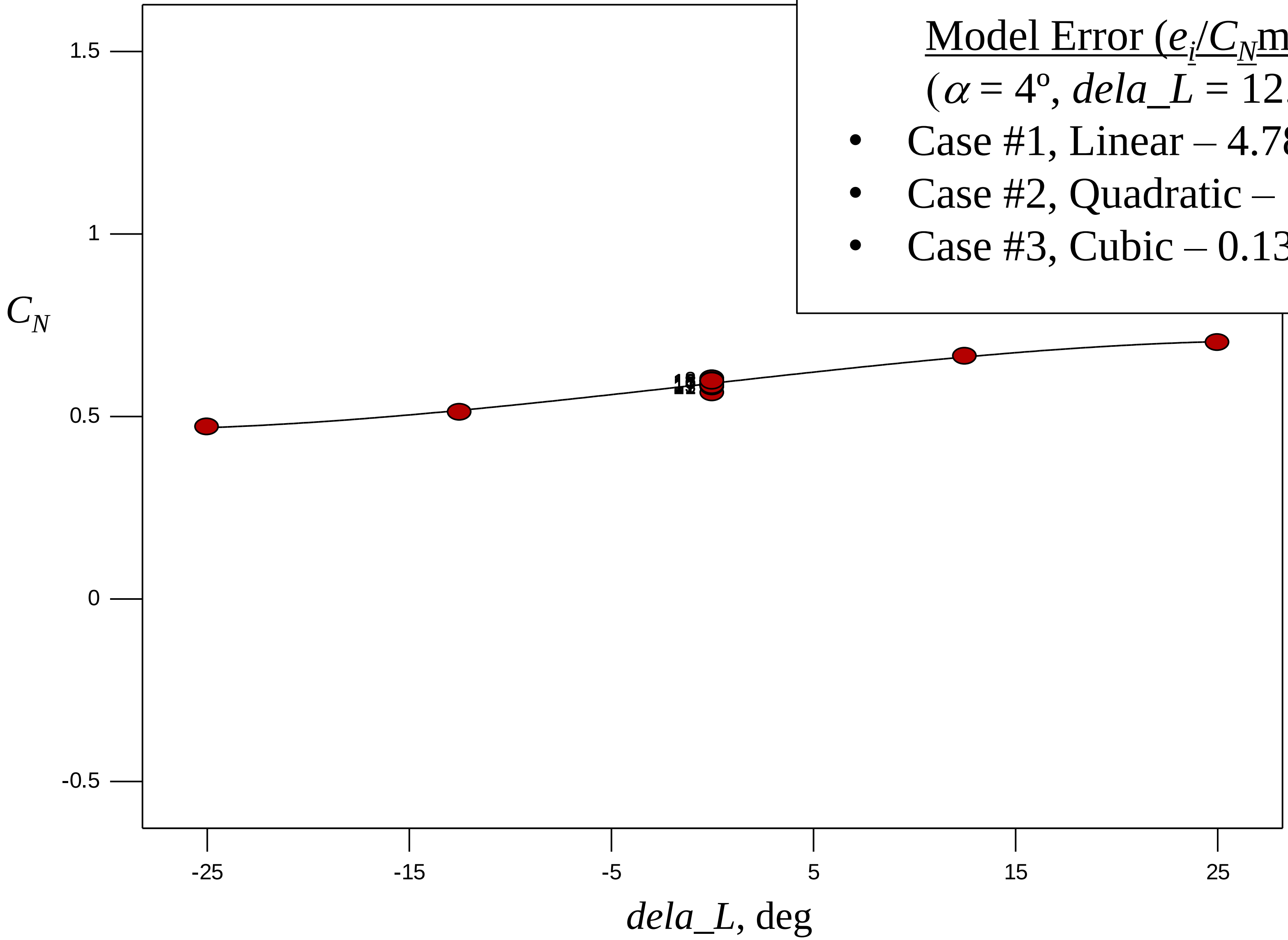


- Residuals vs Run
- Blocks 1-3, opt.
- C_N low α range
- Case #3 model
- Similar final stats

$\pm$3 % error

Model confirmed by validation test; 6 points fail 3% error test

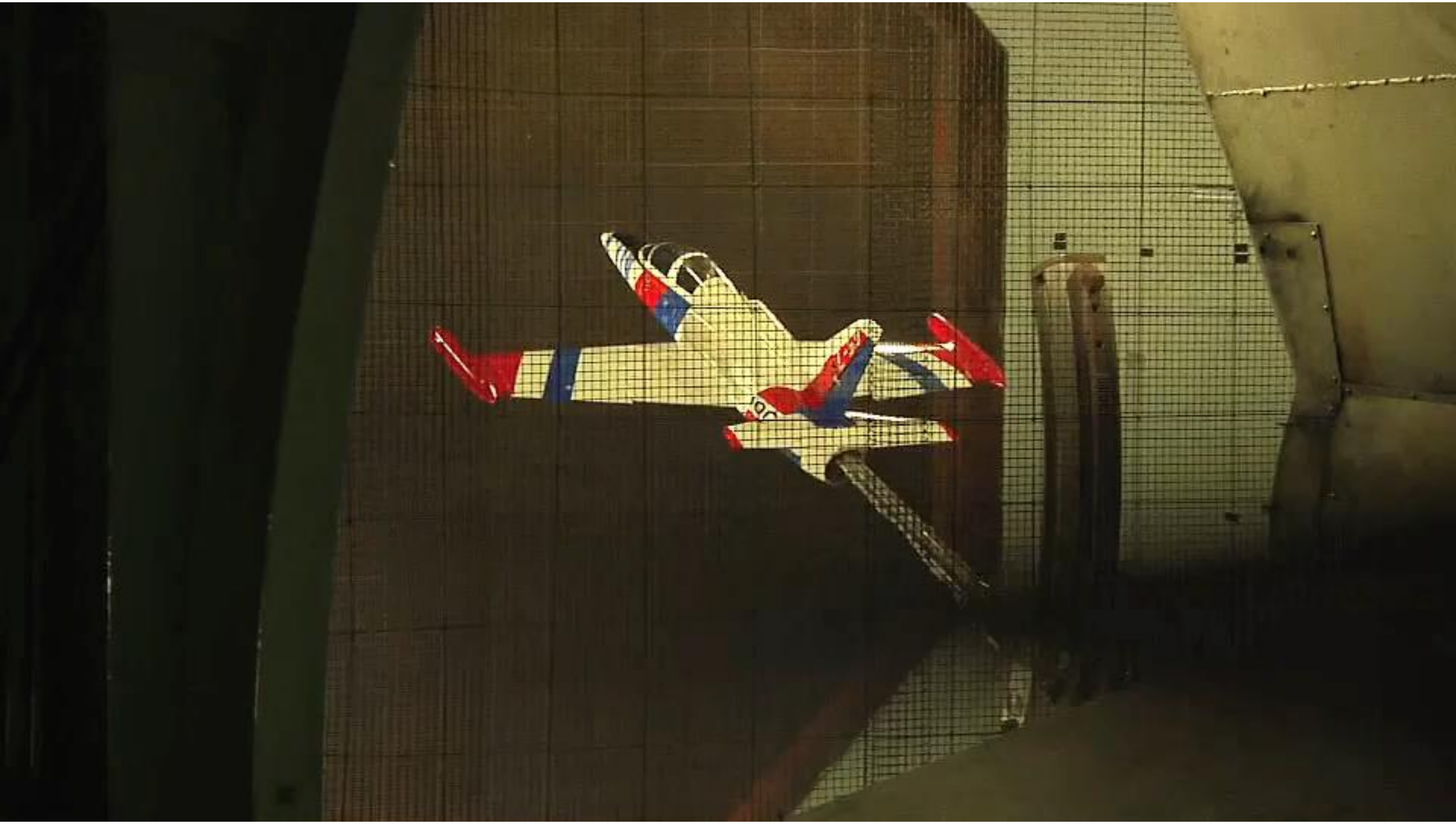
Source of Cubic Terms (DOE blocks 1-3)



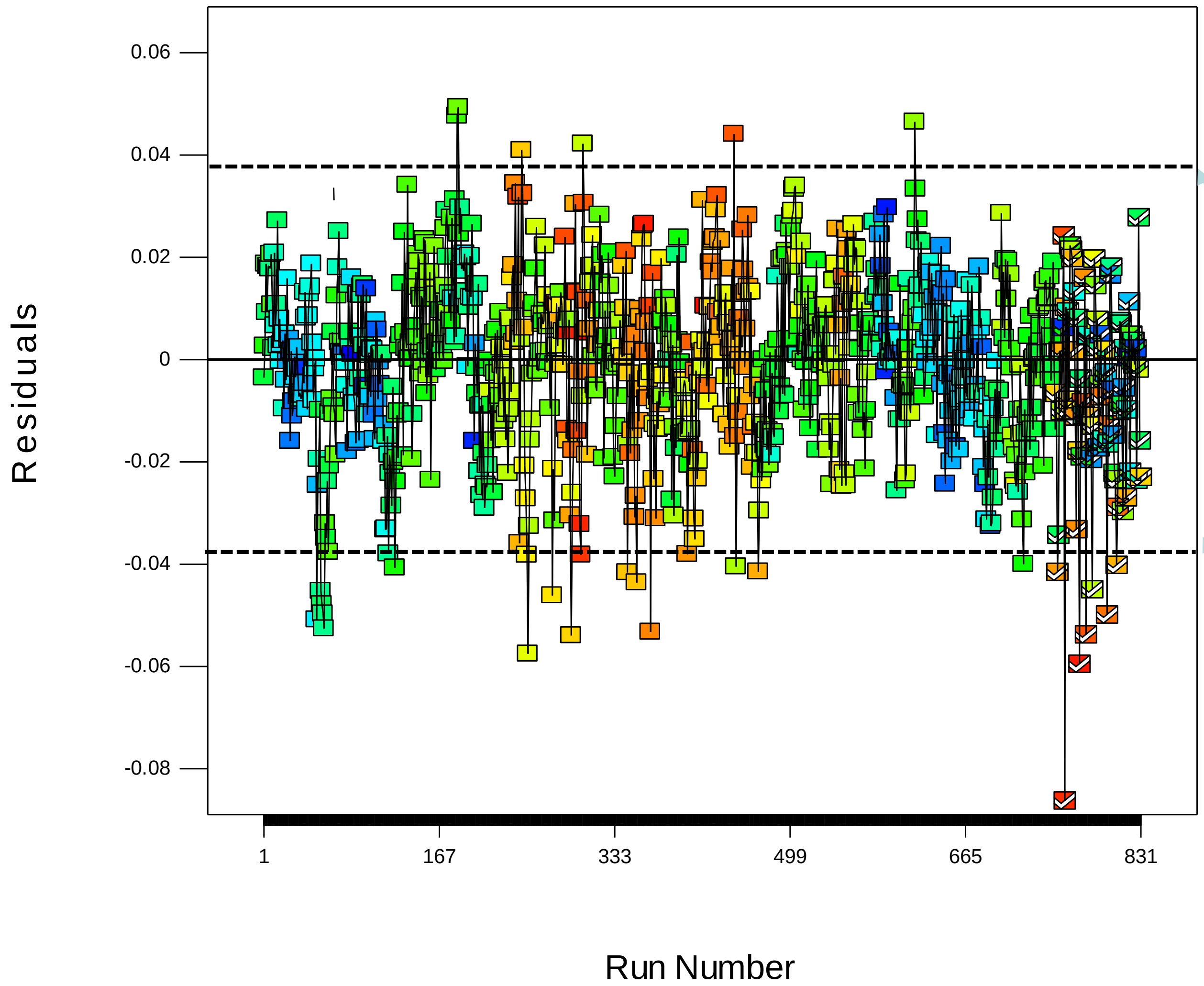
Model Error (e_i/C_Nmax)
(α = 4º, *dela_L* = 12.5º)

- Case #1, Linear – 4.78%
- Case #2, Quadratic – 1.52%
- Case #3, Cubic – 0.13%

L2F Test in LaRC 12-Foot Tunnel



Validation Test, L2F

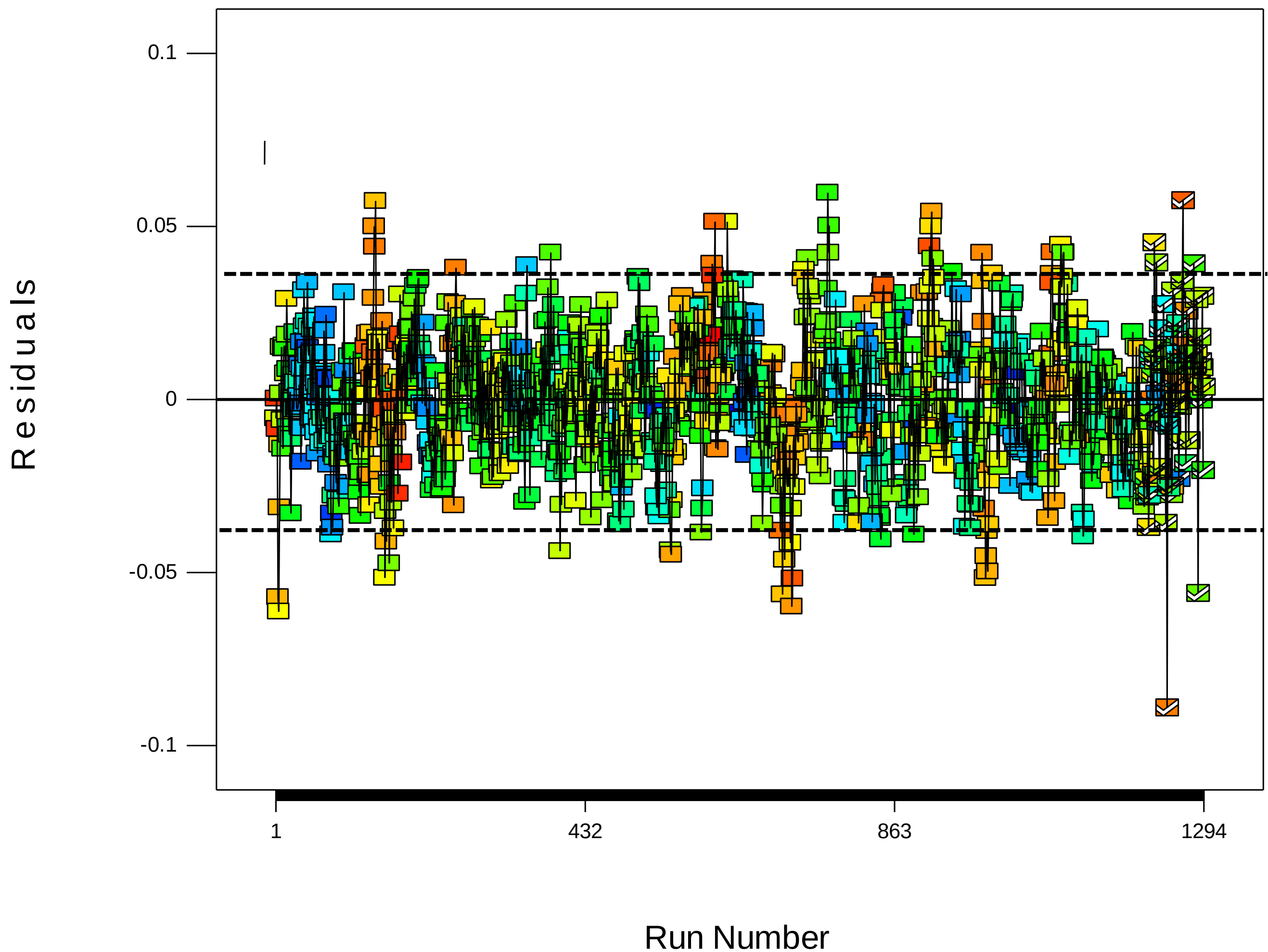


- Residuals vs Run
- Block L2F
- C_N low α range
- Case #3 model
- Similar final stats

$\pm$3 % error

Model confirmed by validation test; 7 points fail 3% error test

Validation Test, Blended DOE-L2F



- Residuals vs Run
- Blocks 1-4
- C_N low α range
- Case #3 model
- Similar final stats

$\pm$3 % error

Model confirmed by validation test; 6 points fail 3% error test

Concluding Remarks

- Sequential testing & validation recommended
  - Obtain data sequentially as required
  - Apply validation test after each block of data
- Efficient test methods demonstrated
  - DOE & L2F approaches provide methods to increase efficiency
  - Blending DOE-L2F
    - Currently a “work in progress” but shows promise
    - Presents a challenge in design phase to combine LBW+HBW factors
- Future Test Refinements
  - Fewer blocks required with continuous sampling
  - Smaller optimal blocks
  - Lower sample rates for “quasi-static” tests
  - For “quasi-static” case, lower bandwidth of HBW inputs
  - Design must reflect significant data added by HBW factors

Questions?

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"All models are wrong, but some are useful" – George E. P. Box